

# Study of Light Scalars, the learned Lessons

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## Abstract

Attention is paid to the production mechanisms of the light scalars that reveal their nature.

In the linear sigma model it is revealed the chiral shielding of the  $\sigma(600)$  meson and shown that the  $\sigma$  field is described by its four-quark component.

The  $\pi\pi$  scattering amplitude is constructed taking into account the  $\sigma(600)$  and  $f_0(980)$  mesons, the chiral shielding of  $\sigma(600)$ , the  $\sigma(600)$ - $f_0(980)$  mixing, and results, obtained on the base of the chiral expansion and the Roy equations. The data agree with the four-quark nature of  $\sigma(600)$  and  $f_0(980)$ .

It is shown that the kaon loop mechanism of the  $\phi$  radiative decays into the light scalar mesons, which is ratified by experiment, is the four-quark transition and points to the four-quark nature of the light scalars.

It is shown also, that the light scalars are produced in the two photon collisions via four-quark transitions in contrast to the classic  $P$  wave tensor  $q\bar{q}$  mesons, which are produced via two-quark transitions  $\gamma\gamma \rightarrow q\bar{q}$ , that points to the four-quark nature of the light scalar mesons, too.

A programme of further investigations is laid down.

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## I. INTRODUCTION [1]

Arising 50 years ago from the linear sigma model (LSM) [1], the light scalar meson problem became central in the nonperturbative QCD for LSM could be its low energy realization. The scalar channels in the region up to 1 GeV is a stumbling block of QCD. The point is that not only perturbation theory fails here, but sum rules as well in view of the fact that isolated resonances are absent in this region.

## II. QCD, CHIRAL LIMIT, CONFINEMENT, $\sigma$ MODELS

$$L = -(1/2)Tr(G_{\mu\nu}(x)G^{\mu\nu}(x)) + \bar{q}(x)(i\hat{D} - M)q(x).$$

$M$  mixes left and right spaces  $q_L(x)$  and  $q_R(x)$ . But in chiral limit  $M \rightarrow 0$  these spaces separate realizing  $U_L(3) \times U_R(3)$  symmetry accurate within violation through gluonic anomaly.

As Experiment suggests, Confinement forms colourless observable hadronic fields and spontaneous breaking of chiral symmetry with massless pseudoscalar fields.

There are two possible scenarios for QCD at low energy.

1.  $U_L(3) \times U_R(3)$  non-linear  $\sigma$ -model.
2.  $U_L(3) \times U_R(3)$  linear  $\sigma$ -model.

The experimental nonet of the light scalar mesons suggests  $U_L(3) \times U_R(3)$  linear  $\sigma$ -model.

## III. HISTORY OF LIGHT SCALAR MESONS

Hunting the light  $\sigma$  and  $\kappa$  mesons had begun in the sixties already. But long-standing unsuccessful attempts to prove their existence in a conclusive way entailed general disappointment and a preliminary information on these states disappeared from Particle Data Group (PDG) Reviews. One of principal reasons against the  $\sigma$  and  $\kappa$  mesons was the fact that both  $\pi\pi$  and  $\pi K$  scattering phase shifts do not pass over  $90^\circ$  at putative resonance masses [2].

#### IV. $SU_L(2) \times SU_R(2)$ LINEAR $\sigma$ MODEL [1, 3, 4]

Situation changes when we showed (1994) that in the linear  $\sigma$  model

$$L = \frac{1}{2} [(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2] - \frac{m_\sigma^2}{2} \sigma^2 - \frac{m_\pi^2}{2} \vec{\pi}^2 - \frac{m_\sigma^2 - m_\pi^2}{8f_\pi^2} \left[ (\sigma^2 + \vec{\pi}^2)^2 + 4f_\pi \sigma (\sigma^2 + \vec{\pi}^2) \right]^2$$

there is a negative background phase which hides the  $\sigma$  meson [3]. It has been made clear that shielding wide lightest scalar mesons in chiral dynamics is very natural.

This idea was picked up and triggered new wave of theoretical and experimental searches for the  $\sigma$  and  $\kappa$  mesons.

##### A. Our approximation [3, 4]

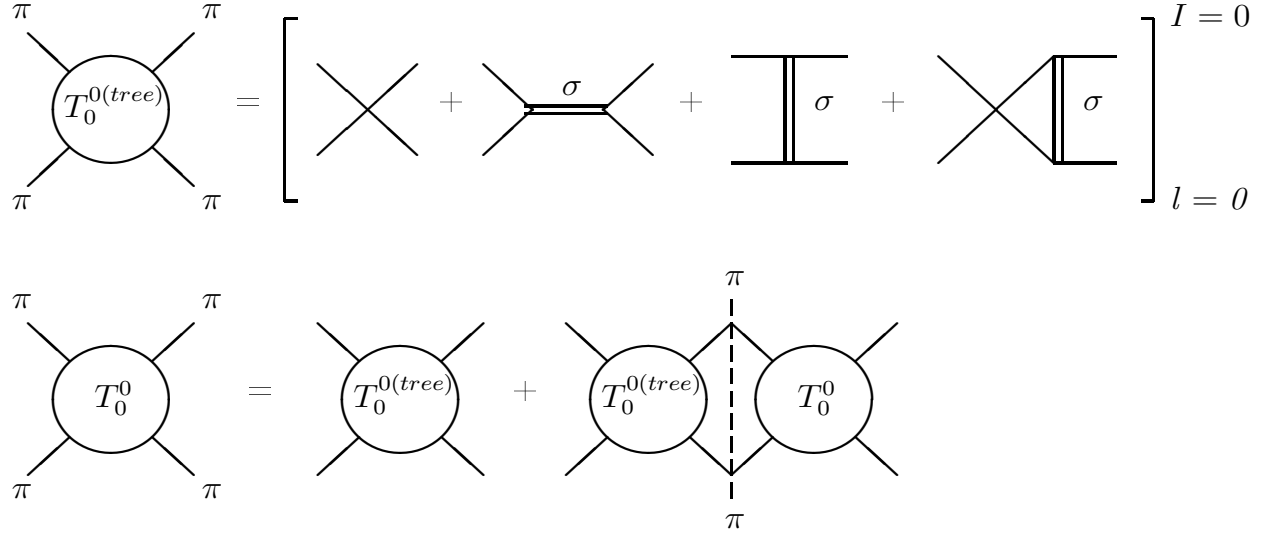


Figure 1: Our approximation.

##### B. Chiral shielding in $\pi\pi \rightarrow \pi\pi$ in our approximation [3, 4]

$$m_\sigma = 0.93 \text{ GeV}, \quad M_{res} = 0.43 \text{ GeV}, \quad \Gamma_{res}^{renorm}(M_{res}^2) = 0.53 \text{ GeV}$$

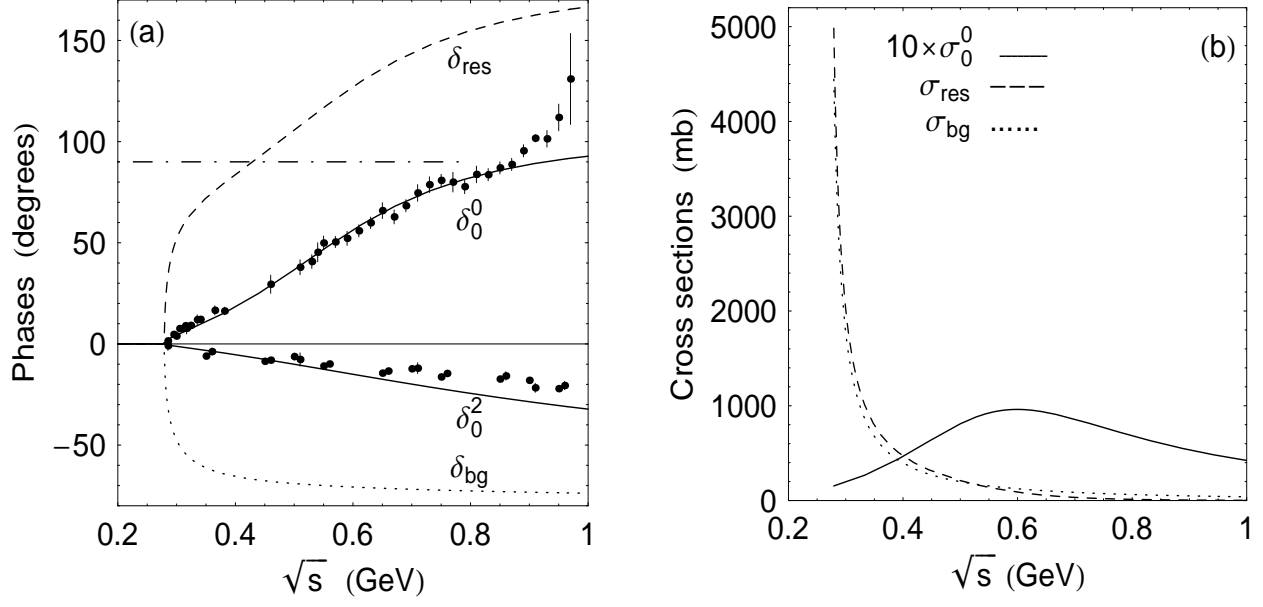


Figure 2: The  $\sigma$  model. Our approximation.  $\delta_0^0 = \delta_{res} + \delta_{bg}$ .

### C. The $\sigma$ pole in $\pi\pi \rightarrow \pi\pi$ in our approximation [4]

$$T_0^0 \rightarrow \frac{g_\pi^2}{s - s_\sigma}, \quad g_\pi^2 = (0.12 + i0.21) \text{ GeV}^2,$$

$$s_\sigma = (0.21 - i0.26) \text{ GeV}^2, \quad \sqrt{s_\sigma} = M_\sigma - i\frac{\Gamma_\sigma}{2} = (0.52 - i0.25) \text{ GeV}.$$

Considering the residue of the  $\sigma$  pole in  $T_0^0$  as the square of its coupling constant to the  $\pi\pi$  channel is not a clear guide to understand the  $\sigma$  meson nature for its great obscure imaginary part.

### D. The $\sigma$ propagator in our approximation [3, 4]

Another thing is the  $\sigma$  propagator

$$\frac{1}{D_\sigma(s)} = \frac{1}{M_{res}^2 - s + \text{Re}\Pi_{res}(M_{res}^2) - \Pi_{res}(s)}. \quad (1)$$

The  $\sigma$  meson self-energy  $\Pi_{res}(s)$  is caused by the intermediate  $\pi\pi$  states, that is, by the four-quark intermediate states. This contribution shifts the Breit-Wigner (BW) mass greatly

$m_\sigma - M_{res} = 0.50$  GeV. So, half the BW mass is determined by the four-quark contribution at least. The imaginary part dominates the propagator modulus in the region  $300 \text{ MeV} < \sqrt{s} < 600 \text{ MeV}$ . So, the  $\sigma$  field is described by its four-quark component at least in this energy (virtuality) region.

## V. TROUBLES AND EXPECTANCIES

In theory the principal problem is impossibility to use the linear  $\sigma$  model in the tree level approximation inserting widths into  $\sigma$  meson propagators because such an approach breaks the both unitarity and the Adler self-consistency condition. The comparison with the experiment requires the non-perturbative calculation of the process amplitudes. Nevertheless, now there are the possibilities to estimate odds of the  $U_L(3) \times U_R(3)$  linear  $\sigma$  model to underlie physics of light scalar mesons in phenomenology, taking into account the idea of chiral shielding of  $\sigma(600)$ , our treatment of  $\sigma(600)$ - $f_0(980)$  mixing based on quantum field theory ideas, Adler's condition, and results, obtained on the base of the chiral expansion and the Roy equations [5].

## VI. PHENOMENOLOGICAL TREATMENT [6], $\delta_0^0 = \delta_B^{\pi\pi} + \delta_{res}$

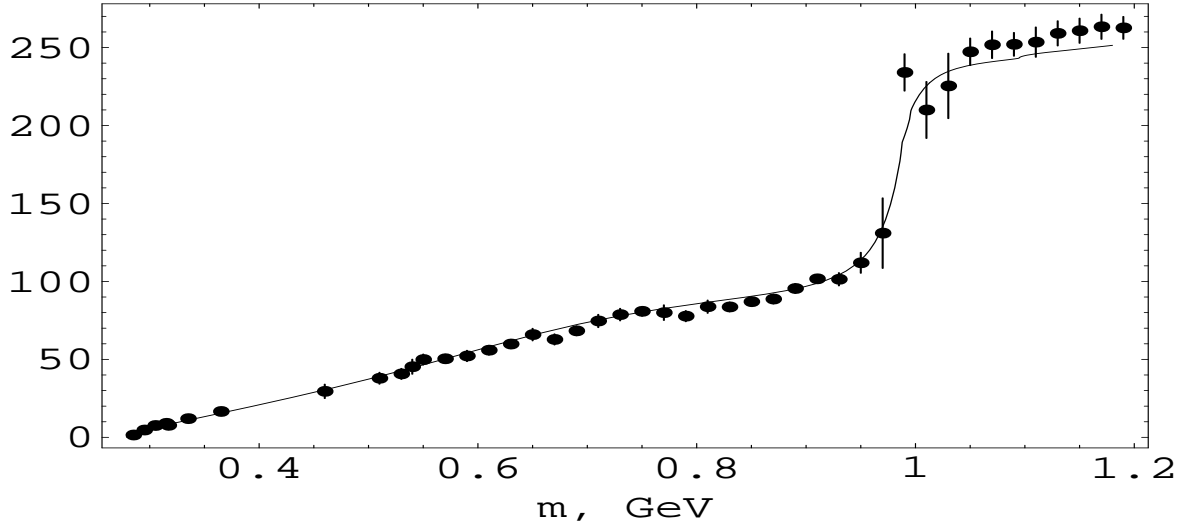


Figure 3:  $\delta_0^0 = \delta_B^{\pi\pi} + \delta_{res}$ .

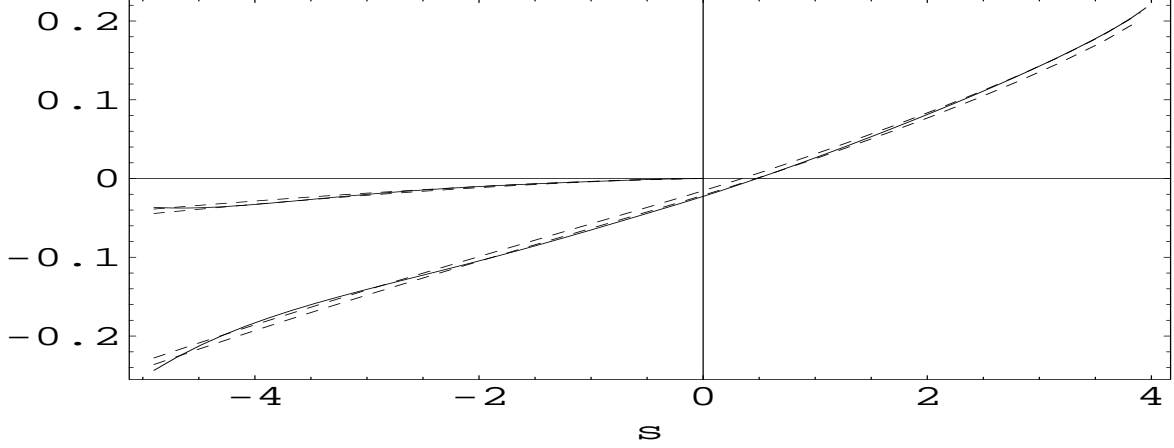


Figure 4:  $T_0^0$  [6], comparison with the calculations based on the chiral expansion and the Roy equations [5],  $s$  in units of  $m_\pi^2$ ; the real part under the threshold:  $-5 < s < 4$ ; the imaginary part on the left cut:  $-5 < s < 0$ .

$$g_{\sigma\pi^+\pi^-}^2/4\pi = 0.57 \text{ GeV}^2, \quad g_{\sigma K^+K^-}^2/4\pi = 0.048 \text{ GeV}^2$$

$$g_{f_0\pi^+\pi^-}^2/4\pi = 0.36 \text{ GeV}^2, \quad g_{f_0K^+K^-}^2/4\pi = 2 \text{ GeV}^2$$

$$m_\sigma = 507 \text{ MeV}, \quad \Gamma_\sigma(m_\sigma) = 353 \text{ MeV}, \quad m_{f_0} = 987 \text{ MeV},$$

$$\Gamma_{f_0}(m_{f_0}) = 130 \text{ MeV}, \quad a_0^0 = 0.226 m_{\pi^+}^{-1}$$

#### A. The $\sigma(600)$ poles [6]

$\Pi^{\pi\pi}(s)$	$\Pi^{K\bar{K}}(s)$	$\Pi^{\eta\eta}(s)$	$\Pi^{\eta\eta'}(s)$	$\Pi^{\eta'\eta'}(s)$	Fit
II	I	I	I	I	$613.8 - 221.4i$
II	II	I	I	I	$609.8 - 291.6i$
II	II	II	I	I	$559.4 - 346.6i$
II	II	II	II	I	$569.7 - 410.7i$
II	II	II	II	II	$581.6 - 411.0i$

Table I: The  $\sigma(600)$  poles (MeV) on different sheets of the complex  $s$  plane depending on lists of polarization operators  $\Pi^{ab}(s)$ .

I. Caprini, G. Colangelo, and H. Leutwyler [5]:

$$\sqrt{s_\sigma} = M_\sigma - i\Gamma_\sigma/2 = (441_{-8}^{+16} - i272_{-12.5}^{+9}) \times \text{MeV}.$$

But the Roy equations are approximate, they take into account only the  $\pi\pi$  channel, but the true  $\pi\pi$  scattering amplitude has the multi (infinity)-list Riemannian surface, that can effect the analytical continuation considerably, especially in the wide resonance case.

So, the current activity, aiming extremely precise determination of the  $\sigma(600)$  pole position, has taken the forms of the Swift's grotesque. Really, the residue of the  $\sigma$  pole can not be connected to coupling constant in the Hermitian (or quasi-Hermitian), see Subsection IV.C and Ref. [4], for it has a large imaginary part and this pole can not be interpreted as a physical state for its huge width.

### B. The $f_0(980)$ poles [6]

$\Pi^{\pi\pi}(s)$	$\Pi^{K\bar{K}}(s)$	$\Pi^{\eta\eta}(s)$	$\Pi^{\eta\eta'}(s)$	$\Pi^{\eta'\eta'}(s)$	Fit
II	I	I	I	I	$990.5 - 19.4 i$
II	II	I	I	I	$1183.2 - 518.6 i$
II	II	II	I	I	$1366.0 - 756.5 i$
II	II	II	II	I	$1390.7 - 813.0 i$
II	II	II	II	II	$1495.6 - 1057.7 i$

Table II: The  $f_0(980)$  poles (MeV) on different sheets of the complex  $s$  plane depending on lists of polarization operators  $\Pi^{ab}(s)$ .

The futility of the approach that is based on the poles treatment may be additionally illustrated by Fit. As seen on line 1 of Table II, the real part of the  $f_0(980)$  pole  $ReM_{f_0}$  on the II sheet of the  $T_0^0$  exceeds the  $K^+K^-$  threshold (987.4 MeV), it means that  $ImM_{f_0}$  equals to  $-(\Gamma(f_0 \rightarrow \pi\pi) - \Gamma(f_0 \rightarrow K^+K^-))/2$ , which is physically meaningless. In this case we should take  $\Pi^{K^+K^-}$  on the second sheet, this gives the pole at  $M_{f_0} = (989.6 - 168.7i) \text{ MeV}$ , with  $ReM_{f_0}$  between the  $K^+K^-$  and  $K^0\bar{K}^0$  thresholds again. But, the analytical properties are specified on the  $s$  plane, and we must consider not  $M_{f_0}$ , but  $M_{f_0}^2 = (0.951 - 0.334 i) \text{ GeV}^2$ . So, we have the pole with a real part below the  $K^+K^-$  (0.975  $\text{GeV}^2$ ) and  $K^0\bar{K}^0$  thresholds, and an imaginary part dictated by analytical continuation of the kaon polarization operators.

## VII. FOUR-QUARK MODEL [7–9]

The nontrivial nature of the well-established light scalar resonances  $f_0(980)$  and  $a_0(980)$  is no longer denied practically anybody. As for the nonet as a whole, even a cursory look at PDG Review gives an idea of the four-quark structure of the light scalar meson nonet,  $\sigma(600)$ ,  $\kappa(800)$ ,  $f_0(980)$ , and  $a_0(980)$ , inverted in comparison with the classical  $P$  wave  $q\bar{q}$  tensor meson nonet,  $f_2(1270)$ ,  $a_2(1320)$ ,  $K_2^*(1420)$ ,  $\phi'_2(1525)$ . Really, while the scalar nonet cannot be treated as the  $P$  wave  $q\bar{q}$  nonet in the naive quark model, it can be easily understood as the  $q^2\bar{q}^2$  nonet, where  $\sigma$  has no strange quarks,  $\kappa$  has the  $s$  quark,  $f_0$  and  $a_0$  have the  $s\bar{s}$  pair. Similar states were found by Jaffe in 1977 in the MIT bag [7].

## VIII. RADIATIVE DECAYS OF $\phi$ -MESON [10]

Ten years later (1987, 1989) we showed that  $\phi \rightarrow \gamma a_0 \rightarrow \gamma\pi\eta$  and  $\phi \rightarrow \gamma f_0 \rightarrow \gamma\pi\pi$  can shed light on the problem of  $a_0(980)$  and  $f_0(980)$  mesons [10]. Now these decays are studied not only theoretically but also experimentally. The first measurements (1998, 2000) were reported by SND and CMD-2. After (2002) they were studied by KLOE in agreement with the Novosibirsk data but with a considerably smaller error.

Note that  $a_0(980)$  is produced in the radiative  $\phi$  meson decay as intensively as  $\eta'(958)$  containing  $\approx 66\%$  of  $s\bar{s}$ , responsible for  $\phi \approx s\bar{s} \rightarrow \gamma s\bar{s} \rightarrow \gamma\eta'(958)$ . It is a clear qualitative argument for the presence of the  $s\bar{s}$  pair in the isovector  $a_0(980)$  state, i.e., for its four-quark nature.

### A. The $K^+K^-$ -loop model [6, 10–14]

When basing the experimental investigations, we suggested [10] the kaon-loop model  $\phi \rightarrow K^+K^- \rightarrow \gamma(a_0/f_0)$ , Fig. 5.

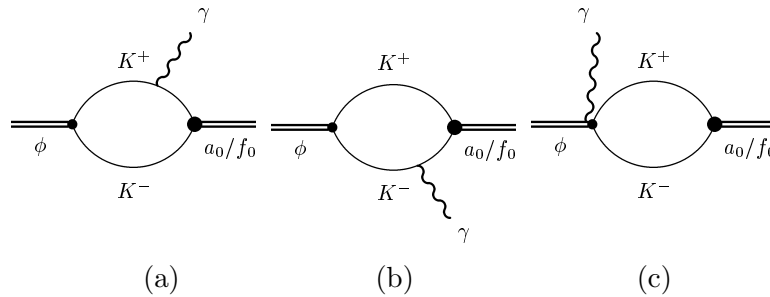


Figure 5: The kaon loop model [10].



This model is used in the data treatment and is ratified by experiment.

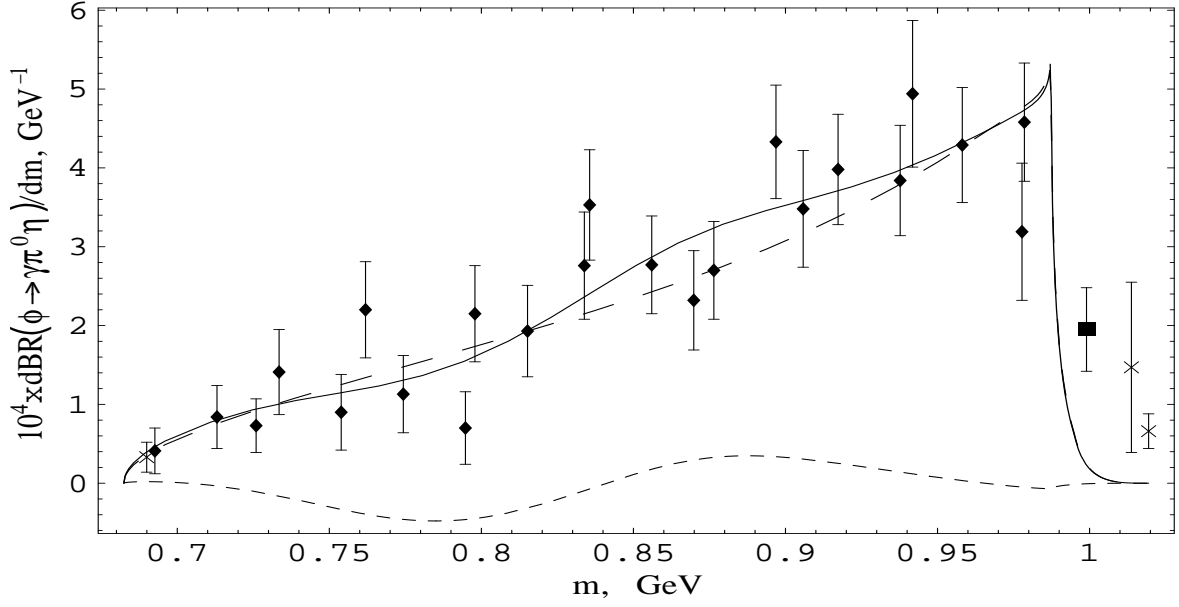


Figure 6: The KLOE data on  $\phi \rightarrow \gamma\pi^0\eta$ , the theory [12].

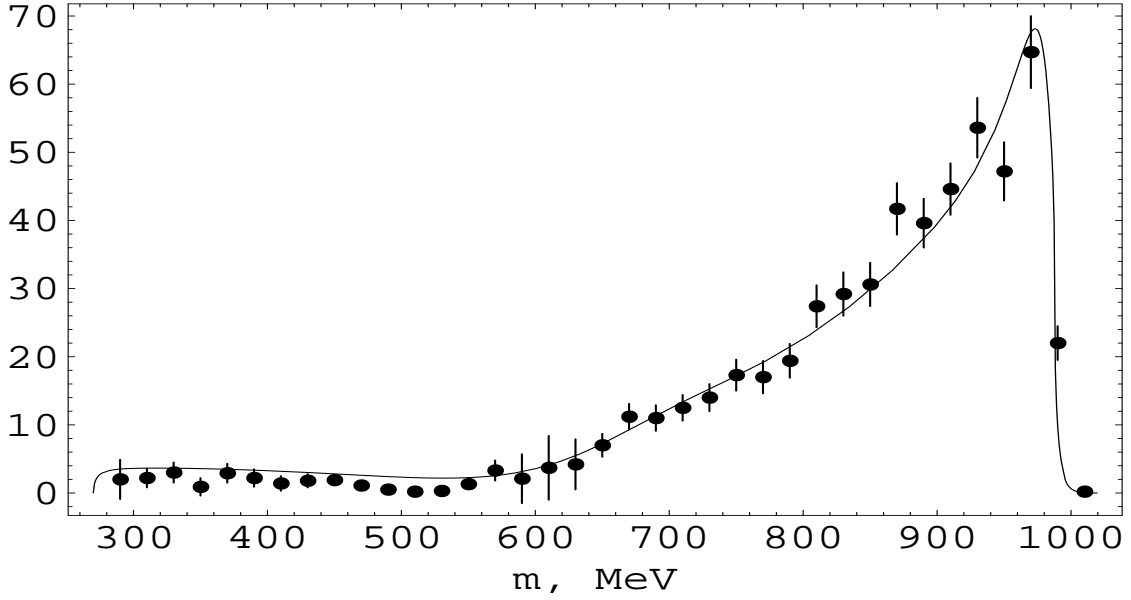


Figure 7: The KLOE data on  $\phi \rightarrow \gamma\pi^0\pi^0$ , the theory [6].

Gauge invariance gives the conclusive arguments in favor of the  $K^+K^-$  - loop transition as the principal mechanism of  $a_0(980)$  and  $f_0(980)$  production in the  $\phi$  radiative decays [13].

The point is to describe the experimental spectra one should to stop the rapid growth of the  $\omega(m)^3$  function, arising of gauge invariance and the phase space factor, at  $\omega(990 \text{ MeV}) = 29 \text{ MeV}$ , where  $\omega(m)$  is the photon energy.

The  $K^+K^-$ -loop model  $\phi \rightarrow K^+K^- \rightarrow \gamma R$  solves this problem in the elegant way with the help of the nontrivial threshold phenomenon.

So, the mechanism of production of  $a_0(980)$  and  $f_0(980)$  mesons in the  $\phi$  radiative decays is established at a physical level of proof: **WE ARE DEALING WITH THE FOUR-QUARK TRANSITION.**

A radiative four-quark transition between two  $q\bar{q}$  states requires creation and annihilation of an additional  $q\bar{q}$  pair, i.e., such a transition is forbidden according to the OZI rule, while a radiative four-quark transition between  $q\bar{q}$  and  $q^2\bar{q}^2$  states requires only creation of an additional  $q\bar{q}$  pair, i.e., such a transition is allowed according to the OZI rule. The large  $N_C$  expansion supports this conclusion [13].

## IX. $a_0(980)/f_0(980) \rightarrow \gamma\gamma$ AND $q^2\bar{q}^2$ -MODEL [15]

Twenty nine years ago (1982) we predicted the suppression of  $a_0(980) \rightarrow \gamma\gamma$  and  $f_0(980) \rightarrow \gamma\gamma$  in the  $q^2\bar{q}^2$  MIT model,  $\Gamma(a_0(980) \rightarrow \gamma\gamma) \sim \Gamma(f_0(980) \rightarrow \gamma\gamma) \sim 0.27 \text{ keV}$  [15].

Experiment supported this prediction

$$\Gamma(a_0 \rightarrow \gamma\gamma) = (0.19 \pm 0.07_{-0.07}^{+0.1})/B(a_0 \rightarrow \pi\eta) \text{ keV, Crystal Ball}$$

$$\Gamma(a_0 \rightarrow \gamma\gamma) = (0.28 \pm 0.04 \pm 0.1)/B(a_0 \rightarrow \pi\eta) \text{ keV, JADE.}$$

$$\Gamma(f_0 \rightarrow \gamma\gamma) = (0.31 \pm 0.14 \pm 0.09) \text{ keV, Crystal Ball,}$$

$$\Gamma(f_0 \rightarrow \gamma\gamma) = (0.24 \pm 0.06 \pm 0.15) \text{ keV, MARK II.}$$

When in the  $q\bar{q}$  model it was anticipated

$$\Gamma(a_0 \rightarrow \gamma\gamma) = (1.5 - 5.9)\Gamma(a_2 \rightarrow \gamma\gamma) = (1.5 - 5.9)(1.04 \pm 0.09) \text{ keV.}$$

$$\Gamma(f_0 \rightarrow \gamma\gamma) = (1.7 - 5.5)\Gamma(f_2 \rightarrow \gamma\gamma) = (1.7 - 5.5)(2.8 \pm 0.4) \text{ keV.}$$

## X. NATURE OF LIGHT SCALAR MESONS AND THEIR PRODUCTION MECHANISMS IN $\gamma\gamma$ COLLISIONS [16–22]

Recently the experimental investigations have made great qualitative advance. The Belle Collaboration published data on  $\gamma\gamma \rightarrow \pi^+\pi^-$  (2007),  $\gamma\gamma \rightarrow \pi^0\pi^0$  (2008), and  $\gamma\gamma \rightarrow \pi^0\eta$  (2009), whose statistics are huge. They not only proved the theoretical expectations based on the four-quark nature of the light scalar mesons, but also have allowed to elucidate the principal mechanisms of these processes.

Specifically, the direct coupling constants of the  $\sigma(600)$ ,  $f_0(980)$ , and  $a_0(980)$  resonances with the  $\gamma\gamma$  system are small with the result that their decays in the two photon are the four-quark transitions caused by the rescatterings  $\sigma \rightarrow \pi^+\pi^- \rightarrow \gamma\gamma$ ,  $f_0(980) \rightarrow K^+K^- \rightarrow \gamma\gamma$ , and  $a_0(980) \rightarrow K^+K^- + \pi^0\eta \rightarrow \gamma\gamma$  in contrast to the two-photon decays of the classic  $P$  wave tensor  $q\bar{q}$  mesons  $a_2(1320)$ ,  $f_2(1270)$  and  $f_2'(1525)$ , which are caused by the direct two-quark transitions  $q\bar{q} \rightarrow \gamma\gamma$  in the main. As a result the practically model-independent prediction of the  $q\bar{q}$  model  $g_{f_2\gamma\gamma}^2 : g_{a_2\gamma\gamma}^2 = 25 : 9$  agrees with experiment rather well.

The two-photon light scalar widths averaged over resonance mass distributions  $\langle \Gamma_{f_0 \rightarrow \gamma\gamma} \rangle_{\pi\pi} \approx 0.19$  keV,  $\langle \Gamma_{a_0 \rightarrow \gamma\gamma} \rangle_{\pi\eta} \approx 0.3$  keV and  $\langle \Gamma_{\sigma \rightarrow \gamma\gamma} \rangle_{\pi\pi} \approx 0.45$  keV. As to the ideal  $q\bar{q}$  model prediction  $g_{f_0\gamma\gamma}^2 : g_{a_0\gamma\gamma}^2 = 25 : 9$ , it is excluded by experiment.

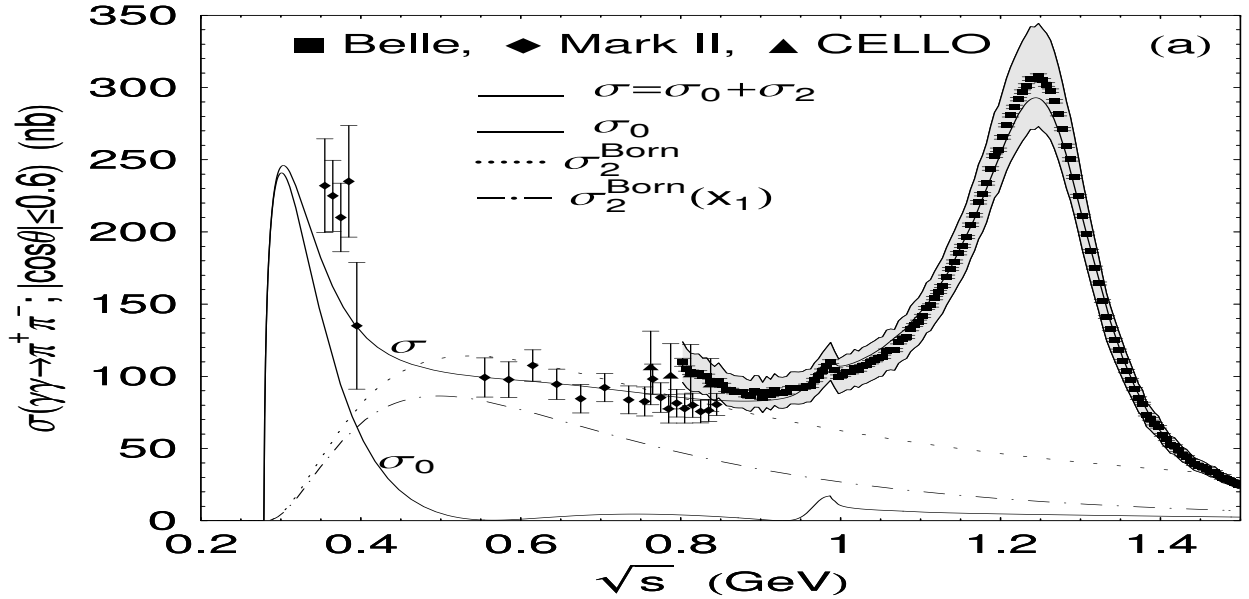


Figure 8: The Belle data on  $\gamma\gamma \rightarrow \pi^+\pi^-$  [17], the theory [20, 22].

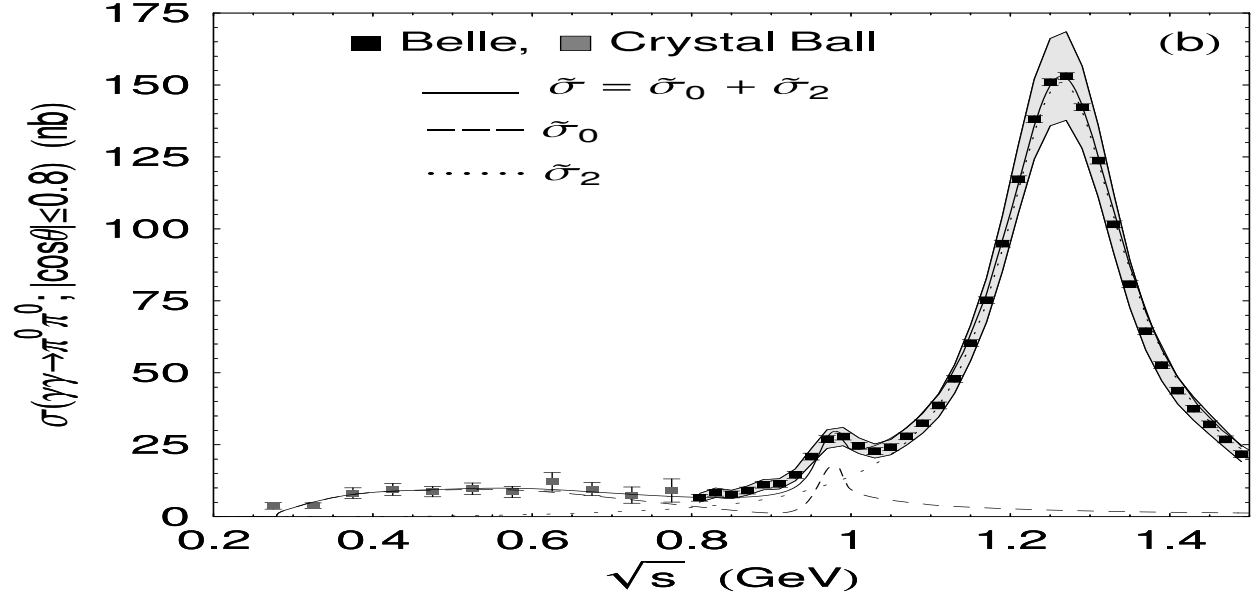


Figure 9: The Belle data on  $\gamma\gamma \rightarrow \pi^0\pi^0$  [18], the theory [20, 22].

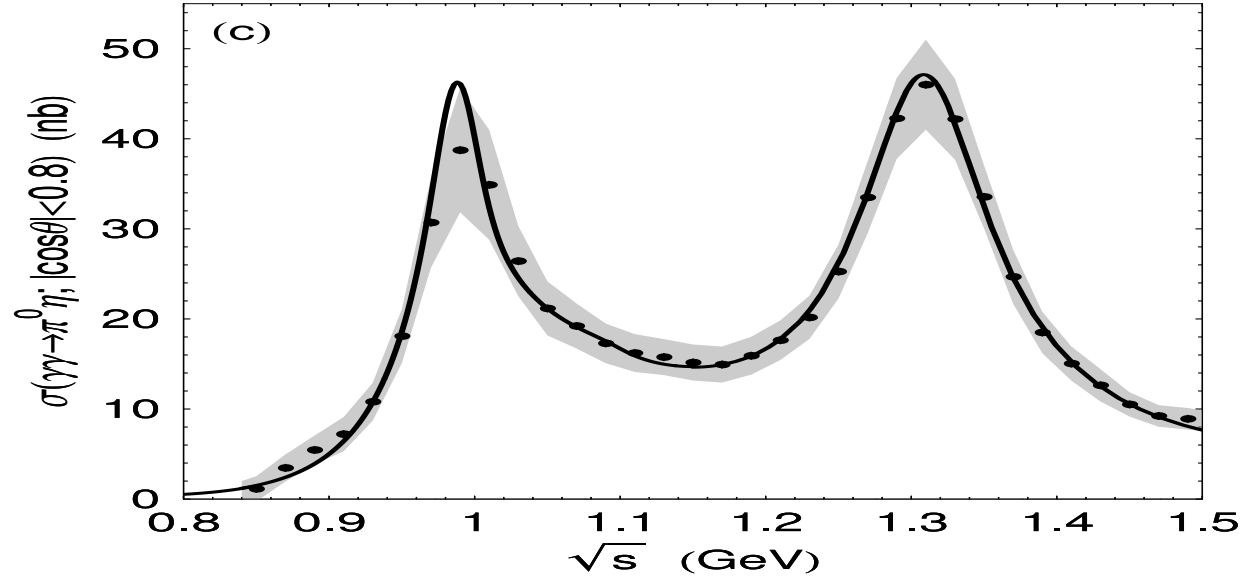


Figure 10: The Belle data on  $\gamma\gamma \rightarrow \pi^0\eta$  [19], the theory [21, 22].

## XI. LESSONS

The mass spectrum of the light scalars,  $\sigma(600)$ ,  $\kappa(800)$ ,  $f_0(980)$ ,  $a_0(980)$ , gives an idea of their  $q^2\bar{q}^2$  structure.

Both intensity and mechanism of the  $a_0(980)/f_0(980)$  production in the radiative decays of  $\phi(1020)$ , the  $q^2\bar{q}^2$  transitions  $\phi \rightarrow K^+K^- \rightarrow \gamma[a_0(980)/f_0(980)]$ , indicate their  $q^2\bar{q}^2$  nature.

Both intensity and mechanism of the scalar meson decays into  $\gamma\gamma$ , basically the four-quark transitions,  $\sigma(600) \rightarrow \pi^+\pi^- \rightarrow \gamma\gamma$ ,  $f_0(980) \rightarrow K^+K^- \gamma\gamma$ , and  $a_0(980) \rightarrow K^+K^- + \pi^0\eta \rightarrow \gamma\gamma$ , indicate their  $q^2\bar{q}^2$  nature, too.

In addition, the absence of the  $J/\psi \rightarrow \gamma f_0(980)$ ,  $a_0(980)\rho$ ,  $f_0(980)\omega$  decays in contrast to the intensive the  $J/\psi \rightarrow \gamma f_2(1270)$ ,  $\gamma f_2'(1525)$ ,  $a_2(1320)\rho$ ,  $f_2(1270)\omega$  decays intrigues against the  $P$  wave  $q\bar{q}$  structure of  $a_0(980)$  and  $f_0(980)$  also.

## XII. OUTLOOK

1.  $\gamma\gamma \rightarrow K^+K^-$ ,  $K^0\bar{K}^0$  near the thresholds, it is expected a drastic suppression of the Born contribution in the  $K^+K^-$  channel.  $\gamma\gamma^*(Q^2) \rightarrow \pi^0\pi^0$ ,  $\pi^0\eta$ , it is expected a drastic decrease of the  $\sigma(600)$ ,  $f_0(980)$  and  $a_0(980)$  contributions with increasing  $Q^2$  as opposed to a decrease of the  $f_2(1270)$  and  $a_2(1320)$  ones.
2. Search for  $J/\psi \rightarrow f_0(980)\omega$  and  $J/\psi \rightarrow a_0(980)\rho$ .
3. Search for the  $a_0(980) - f_0(980)$  mixing [23] in
  - i)  $J/\psi \rightarrow f_0(980)\phi \rightarrow a_0(980)\phi \rightarrow \pi^0\eta\phi$  [24, 25] and
  - ii)  $\pi^-p \rightarrow f_0(980)n \rightarrow a_0(980)n \rightarrow \pi^0\eta n$  [26],
 here it is expected a strong jump in the spin asymmetry that could give an exclusive information on  $(g_{a_0K^+K^-} \cdot g_{f_0K^+K^-})/4\pi$ .
4. The new precise experiment on  $\pi\pi \rightarrow K\bar{K}$  would give the crucial information about the inelasticity  $\eta_0^0$  and about the phase  $\delta_B^{K\bar{K}}(m)$  near the  $K\bar{K}$  threshold. The precise measurement of the inelasticity  $\eta_0^0$  near 1 GeV in  $\pi\pi \rightarrow \pi\pi$  would also be very important.

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